



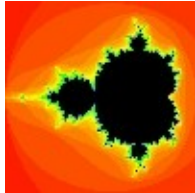
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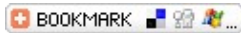
September 2006

Features



## Unveiling the Mandelbrot set

by Robert L. Devaney



### A brief introduction to complex numbers

Complex numbers are based on the number  $i$  which is defined to be the square root of  $-1$ , so  $i$  times  $i$  equals  $-1$ . This number isn't a real number, in other words it does not appear on the usual number line. For this reason it is called an *imaginary number*, a slightly contentious name. Now any complex number is of the form  $a + ib$ , where  $a$  and  $b$  are ordinary real numbers. The numbers  $1 + i2$  or  $5 - i8$  are both complex numbers.

Complex numbers are added (as you would expect) like this:

$$(a + ib) + (c + id) = (a + c) + i(b + d),$$

and multiplied (again as you would expect) like this:

$$(a + ib)(c + id) = ac + iad + ibc + i2bd = ac - bd + i(ad + bc).$$

You can apply a function  $x^2 + c$  even when the seed  $x_0$  and the constant  $c$  are complex numbers: if  $x = a + ib$  and  $c = s + it$  then

$$x^2 + c = (a + ib)^2 + (s + it) = a^2 - b^2 + i(2ab) + s + it = (a^2 - b^2 + s) + i(2ab + t),$$

## Unveiling the Mandelbrot set

which is a new complex number.

Unless a complex number  $a + ib$  has  $b = 0$ , we cannot find it on the ordinary number line. We can, however, visualise it as a point on the plane: to the number  $a + ib$  simply associate the point with co-ordinates  $(a,b)$ . You can see that the real numbers are contained in the complex numbers: a real number  $x$  seen as a complex number is simply  $x + i0$  and corresponds to the point with co-ordinates  $(x,0)$ .

To summarise, every complex number represents a point on the plane and vice versa. We can visualise the orbit of any seed, including 0, as a sequence of points on the plane.

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