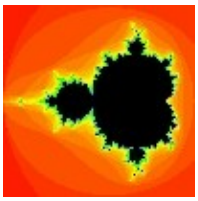
September 2006
Features



Unveiling the Mandelbrot set

by Robert L. Devaney

A brief introduction to complex numbers

Complex numbers are based on the number i which is defined to be the square root of −1, so i times i equals −1. This number isn't a real number, in other words it does not appear on the usual number line. For this reason it is called an *imaginary number*, a slightly contentious name. Now any complex number is of the form $a + ib$, where a and b are ordinary real numbers. The numbers $1 + i2$ or $5 - i8$ are both complex numbers.

Complex numbers are added (as you would expect) like this:

$$(a + ib) + (c + id) = (a + c) + i(b + d),$$

and multiplied (again as you would expect) like this:

$$(a + ib)(c + id) = ac + iad + ibc + i2bd = ac - bd + i(ad + bc).$$

You can apply a function $x^2 + c$ even when the seed x_0 and the constant c are complex numbers: if $x = a + ib$ and $c = s + it$ then

$$x^2 + c = (a + ib)^2 + (s + it) = a^2 - b^2 + i(2ab) + s + it = (a^2 - b^2 + s) + i(2ab + t),$$

which is a new complex number.

Unless a complex number $a + ib$ has $b = 0$, we cannot find it on the ordinary number line. We can, however, visualise it as a point on the plane: to the number $a + ib$ simply associate the point with co−ordinates (a,b). You can see that the real numbers are contained in the complex numbers: a real number x seen as a complex number is simply $x + i0$ and corresponds to the point with co−ordinates $(x,0)$.

To summarise, every complex number represents a point on the plane and vice versa. We can visualise the orbit of any seed, including 0, as a sequence of points on the plane.

Return to the article