
January 1999
Regulars

A Reader's Solution

Here is Tom Holden's solution to puzzle number 6.

When you get your first coin, you are guaranteed to get a coin you have not yet got.

When you get you second coin, there is 1/22 chance of it being one you already have, so there is a 21/22 chance of getting one you do not yet have.

When you have two different coins, there is a 2/22=1/11 chance of getting one you already have, so there is a 20/22=10/11 chance of getting a new one.

When you have n−1 different coins (or when if you get a new coin it will be your n'th coin), there is a (n−1)/22 chance of getting one you already have, so the re is a 1−(n−1)/22 chance of getting a new one.

Now the average total number of coins you need to get before you get the full set is the sum (for i=1 to 22) of the average number of coins you have to get before you get your i'th different coin. So to work out the solution of the problem, we must work out how the average number of coins needed to get the i'th different coin is affected by the number i.

This can be done easily since we know the probability of getting a coin we do not already have is 1−(i−1)/22 and so the expected number of coins we need to take before we get a new coin is 1/(1−(i−1)/2 2)=22/(23−i) and so the total number of coins is the sum (for i=1 to 22) of 22/(23−i)=22 times the sum (for i=1 to 22) of 1/i since changing the order of a sum makes no difference to its result (a+b=b+a).

Now the answer to the problem is given by:

22*(1+1/2+1/3+...+1/22)=81 to the nearest whole number of coins.

So on average, you would have to get 81 coins before you had a whole set of 22.

Plus is part of the family of activities in the Millennium Mathematics Project, which also includes the NRICH and MOTIVATE sites.